For nonideal gases a general relation is

\[
p = MP/RT,
\]

(6.4)

where the compressibility factor \(z\) is correlated empirically in terms of reduced properties \(T/T_c\) and \(P/P_c\) and the acentric factor. This subject is treated for example by Reid et al. (1977, p. 26) and Walas (1985, pp. 17, 70). Many PVT equations of state are available. That of Redlich and Kwong may be written in the form

\[
V = b + RT/(P + a/\sqrt{T}V^2),
\]

(6.5)

which is suitable for solution by direct iteration as used in Example 6.1.

Flow rates are expressible as linear velocities or in volumetric, mass, or weight units. Symbols for and relations between the several modes are summarized in Table 6.1. Other dimensionless groups occur less frequently and will be mentioned as they occur in this chapter; a long list is given in Perry’s Chemical Engineers Handbook (McGraw-Hill, New York, 1984, p. 5.62).

---

**Example 6.1**

**Density of a Nonideal Gas from Its Equation of State**

The Redlich–Kwong equation of carbon dioxide is

\[
(P + 63.72(10^9)\sqrt{T}V^2)(V - 29.664) = 82.05T
\]

with \(P\) in atm, \(V\) in mL/g mol and \(T\) in K. The density will be found at \(P = 20\) and \(T = 400\). Rearrange the equation to

\[
V = 29.664 + (82.05)(400)/(20 + 63.72(10^9)\sqrt{400} V^2).
\]

Substitute the ideal gas volume on the right, \(V = 1641\); then find \(V\) on the left; substitute that value on the right, and continue. The successive values of \(V\) are

\[
V = 1641, 1579, 1572.1, 1571.3, 1571.2, \ldots \text{ mL/g mol}
\]

and converge at 1571.2. Therefore, the density is

\[
\rho = 1/V = 1/1571.2, \text{ or } 0.6365 \text{ g mol/L or } 28.00 \text{ g/L.}
\]
### TABLE 6.1. Flow Quantities, Reynolds Number, and Friction Factor

<table>
<thead>
<tr>
<th>Flow Quantity</th>
<th>Typical Units</th>
<th>Common Units</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>ft/sec</td>
<td>m/sec</td>
<td></td>
</tr>
<tr>
<td>Volumetric</td>
<td>cft/sec</td>
<td>m³/sec</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>lb/sec</td>
<td>kg/sec</td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>lbf/ft²/sec</td>
<td>N/sec</td>
<td></td>
</tr>
<tr>
<td>Mass/area</td>
<td>lb/(sqft)(sec)</td>
<td>kg/m² sec</td>
<td></td>
</tr>
<tr>
<td>Weight/area</td>
<td>lbf/(sqft)(sec)</td>
<td>N/m² sec</td>
<td></td>
</tr>
<tr>
<td>Reynolds Number (with (A = \pi D^2/4))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Re = \frac{Dp}{\nu} = \frac{DP}{\mu} = \frac{4Qp}{\pi D\mu} = \frac{4m}{\pi D\mu})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Friction Factor

\[
f = \frac{\Delta P}{\rho} \left( \frac{D^2}{2z} \right) = 2gD\Delta P/Lp\nu^2 = 1.6364 \left( \frac{0.135c}{D} + 6.5 \right)^{1/2} Re
\]  \(\text{(2)}\)

\[
\frac{\Delta P}{\rho} \left( \frac{D^2}{2z} \right) = \frac{8LQ^2}{g_\alpha^2 D^2} \left( \frac{2Lm^2}{g_\alpha^2 D^2} \right) f = \frac{8LQ^2}{g_\alpha^2 D^2} f = \frac{L^2 Q^2}{2g_\alpha D^2} f
\]  \(\text{(3)}\)

#### In the units

- \(D = \text{in.}, \quad m = \text{lb/hr}\)
- \(Q = \text{cft/sec}, \quad \mu = \text{cp}\)
- \(\rho = \text{specific gravity}\)
- \(Re = \frac{6.314m}{D\mu} = 1.418(10^5)pQ\)
- \(\frac{\Delta P}{L} = \frac{3.663(10^{-5}) pQ}{\rho D^4}, \text{ atm/ft}\)
- \(\frac{\Delta P}{L} = \frac{5.365(10^{-5}) pQ}{\rho D^4}, \text{ psi/ft}\)
- \(\frac{\Delta P}{L} = \frac{0.9973 pQ}{D^2}, \text{ psi/ft}\)

#### Laminar Flow

\(Re < 2300\)

\[
f = \frac{64}{Re}
\]

\[
\frac{\Delta P}{L} = \frac{32 \mu u}{D^2} = \frac{1.841(10^{-7}) \mu u}{\rho D^4}, \text{ atm/ft}
\]

\[
\frac{\Delta P}{L} = \frac{2.707(10^{-6}) \mu u}{\rho D^4}, \text{ psi/ft}
\]

\[
\frac{\Delta P}{L} = \frac{35.083 \mu u}{D^2}, \text{ psi/ft}
\]  \(\text{(7a)}\)

#### Gravitation Constant

- \(g_\alpha = 1 \text{ kg m/N sec}^2\)
- \(= 1 \text{ g cm/dyn sec}^2\)
- \(= 9.806 \text{ kg m/kgf sec}^2\)
- \(= 32.174 \text{ lbm ft/lbf sec}^2\)
- \(= 1 \text{ slug ft/lbf sec}^2\)
- \(= 1 \text{ lbm ft/poundal sec}^2\)

### 6.2. ENERGY BALANCE OF A FLOWING FLUID

The energy terms associated with the flow of a fluid are

1. Elevation potential \((g/g_\alpha)z,\)
2. Kinetic energy, \(u^2/2g_\alpha,\)
3. Internal energy, \(U,\)
4. Work done in crossing the boundary, \(PV,\)
5. Work transfer across the boundary, \(W,\)
6. Heat transfer across the boundary, \(Q.\)

Figure 6.1 represents the two limiting kinds of regions over which energy balances are of interest: one with uniform conditions throughout (completely mixed), or one in plug flow in which gradients are present. With single inlet and outlet streams of a uniform region, the change in internal energy within the boundary is

\[d(mU) = m dU + U dm = m dU + U(dm_1 - dm_2)\]

\[= dQ - dW_1 + [H_1 + u^2/2g_\alpha + (g/g_\alpha)z_1] dm_1\]

\[- [H_2 + u^2/2g_\alpha + (g/g_\alpha)z_2] dm_2. \quad (6.6)\]

One kind of application of this equation is to the filling and emptying of vessels, of which Example 6.2 is an instance.

Under steady state conditions, \(d(mU) = 0\) and \(dm_1 = dm_2 = dm,\) so that Eq. (6.6) becomes

\[\Delta H + \Delta u^2/2g_\alpha + (g/g_\alpha)\Delta z = (Q - W_1)/m. \quad (6.7)\]

or

\[\Delta U + \Delta (PV) + \Delta u^2/2g_\alpha + (g/g_\alpha)\Delta z = (Q - W_1)/m. \quad (6.8)\]

or

\[\Delta U + \Delta (P/\rho) + \Delta u^2/2g_\alpha + (g/g_\alpha)\Delta z = (Q - W_1)/m. \quad (6.9)\]

For the plug flow condition of Figure 6.1(b), the balance is made in terms of the differential changes across a differential length \(dL\) of the vessel, which is

\[dH + (1/g_\alpha)u du + (g/g_\alpha)dz = dQ - dW_1, \quad (6.10)\]

where all terms are per unit mass.

---

**Figure 6.1.** Energy balances on fluids in completely mixed and plug flow vessels. (a) Energy balance on a bounded space with uniform conditions throughout, with differential flow quantities \(dm_1\) and \(dm_2.\) (b) Differential energy balance on a fluid in plug flow in a tube of unit cross section.
EXAMPLE 6.2
Unsteady Flow of an Ideal Gas through a Vessel

An ideal gas at 350 K is pumped into a 1000 L vessel at the rate of 6 g mol/min and leaves it at the rate of 4 g mol/min. Initially the vessel is at 310 K and 1 atm. Changes in velocity and elevation are negligible. The contents of the vessel are uniform. There is no work transfer.

Thermodynamic data:
\[ U = C_v T = 5T, \]
\[ H = C_p T = 7T. \]

Heat transfer:
\[ dQ = h(300 - T) \, d\theta \]
\[ = 15(300 - T) \, d\theta. \]

The temperature will be found as a function of time \( \theta \) with both \( h = 15 \) and \( h = 0. \)
\[ \begin{align*}
  d\theta_1 &= 6 \, d\theta, \\
  d\theta_2 &= 4 \, d\theta, \\
  d\theta &= d\theta_1 - d\theta_2 = 2 \, d\theta, \\
  n_0 &= \rho_0 V / R T_0 = 1000 / (0.08205)(310) = 39.32 \, \text{gmol}, \\
  n &= n_0 + 2\theta, \\
  V = 1000 \ell \\
  T_1 = 350 \\
  n_1 = 6 \\
  dQ \\
  d_4 = 0 \\
  T_2 = 7 \\
  n_2 = 4 \\
\end{align*} \]

Energy balance
\[ d(nU) = n \, dU + U \, dn = nC_v \, dT + C_v (2 \, d\theta) \]
\[ = H_1 \, d\theta_1 - H_2 \, d\theta_2 + dQ - dW, \]
\[ = C_p (6T_1 - 4T) \, d\theta + h(300 - T) \, d\theta. \]

This rearranges into
\[ \int_0^{\theta} \frac{d\theta}{n_0 + 2\theta} = \int_{T_1}^{T_2} \frac{dT}{(1/C_v) \left[ 6C_p T_1 + 300h - (4C_p + 2C_v + h)T \right]} \]
\[ \begin{align*}
  h &= 15, \\
  h &= 0, \\
\end{align*} \]

The integrals are rearranged to find \( T \),
\[ T_2 = \begin{cases} 
  362.26 - 52.26\left( \frac{1}{1 + 0.0509\theta} \right)^{3.3}, & h = 15, \\
  386.84 - 76.84\left( \frac{1}{1 + 0.0509\theta} \right)^{3.8}, & h = 0. \\
\end{cases} \]

Some numerical values are
\[ \begin{array}{cccc}
  \theta & h = 15 & h = 0 \\
  0 & 310 & 310 & 1 & 1 \\
  0.5 & 316.5 & 317.0 & 1.02 & 1.02 \\
  1 & 322.1 & 323.2 & 1.37 & 1.37 \\
  5 & 346.5 & 354.4 & 1.73 & 1.73 \\
  10 & 356.4 & 370.8 & 1.80 & 1.80 \\
\end{array} \]

The pressures are calculated from
\[ P = \frac{nRT}{V} = \frac{(39.32 + 2\theta)(0.08205)T}{1000}. \]

Friction is introduced into the energy balance by noting that it is a mechanical process, \( dW_f \), whose effect is the same as that of an equivalent amount of heat transfer \( dQ_f \). Moreover, the total effective heat transfer results in a change in entropy of the flowing liquid given by
\[ T \, dS = dQ + dW_f. \]

When the thermodynamic equivalent
\[ dH = V \, dP + T \, dS \]
and Eq. (6.11) are substituted into Eq. (6.10), the net result is
\[ V \, dP + (1/g_c) u \, du + (g/g_s) \, dz = -(dW_f + dW_f), \]
which is known as the mechanical energy balance. With the expression for friction of Eq. (6.18) cited in the next section, the mechanical energy balance becomes
\[ V \, dP + (1/g_c) u \, du + (g/g_s) \, dz + \frac{fdL}{2g_c D} = -dW_f, \]

For an incompressible fluid, integration may be performed term by term with the result
\[ \Delta P/p + \Delta u^2/2g_c + (g/g_s) \Delta z = -(W_f + W_f). \]  

The apparent number of variables in Eq. (6.13) is reduced by the substitution \( u = V/A \) for unit flow rate of mass, where \( A \) is the cross-sectional area, so that
\[ V \, dP + (1/g_c A^2) V \, dV + (g/g_s) \, dz = -(dW_f + dW_f). \]

Integration of these energy balances for compressible fluids under several conditions is covered in Section 6.7.

The frictional work loss \( W_f \) depends on the geometry of the system and the flow conditions and is an empirical function that will be explained later. When it is known, Eq. (6.13) may be used to find a net work effect \( W_f \) for otherwise specified conditions.

The first three terms on the left of Eq. (6.14) may be grouped into a single stored energy terms as
\[ \Delta E = \Delta P/p + \Delta u^2/2g_c + (g/g_s) \Delta z, \]
EXAMPLE 6.3
Units of the Energy Balance

In a certain process the changes in stored energy and the friction are

\[ \Delta E = -135 \text{ ft lbf/lb} \]
\[ w_f = 13 \text{ ft lbf/lb}. \]

The net work will be found in several kinds of units:

\[ w_e = -(\Delta E + w_f) = 122 \text{ ft lbf/lb}, \]
\[ w_f = 122 \frac{\text{ft lbf}}{\text{lbf}} \times 4.448 \frac{\text{N m}}{\text{kg f m}} \times \frac{1}{9.806 \text{ N/kg}} = 37.19 \frac{\text{kg f m}}{\text{m}}. \]

At sea level, numerically lbf = lb and kgf = kg.

Accordingly,

\[ w_f = 122 \frac{\text{ft lbf lbf kg f m}}{\text{lbf kg f 3.28 ft}} = 37.19 \frac{\text{kg f m}}{\text{kg}} , \]

as before.

The units of every term in these energy balances are alternately:

- ft lb/lb with \( g_e = 32.174 \) and \( g \) in ft/sec² (32.174 at sea level).
- N m/kg = J/kg with \( g_e = 1 \) and \( g \) in m/sec² (1.000 at sea level).
- kg f/m/kg with \( g_e = 9.806 \) and \( g \) in m/sec² (9.806 at sea level).

Example 6.3 is an exercise in conversion of units of the energy balances.

The sign convention is that work input is a negative quantity and consequently results in an increase of the terms on the left of Eq. (6.17). Similarly, work is produced by the flowing fluid only if the stored energy \( \Delta E \) is reduced.

6.3. LIQUIDS

Velocities in pipe lines are limited in practice because of

1. the occurrence of erosion.
2. economic balance between cost of piping and equipment and the cost of power loss because of friction which increases sharply with velocity.

Although erosion is not serious in some cases at velocities as high as 10–15 ft/sec, conservative practice in the absence of specific knowledge limits velocities to 5–6 ft/sec.

Economic optimum design of piping will be touched on later, but the rules of Table 6.2 of typical linear velocities and pressure drops provide a rough guide for many situations.

The correlations of friction in lines that will be presented are for new and clean pipes. Usually a factor of safety of 20–40% is advisable because pitting or deposits may develop over the years.

For other shapes and annular spaces, \( D \) is replaced by the hydraulic diameter

\[ D_h = 4(\text{cross section})/\text{wetted perimeter}. \]

For an annular space, \( D_h = D_o - D_i \).

In laminar flow the friction is given by the theoretical Poiseuille equation

\[ f = 64/\text{Re}, \text{ } \text{Re} < 2100, \text{ } \text{approximately}. \]

At higher Reynolds numbers, the friction factor is affected by the roughness of the surface, measured as the ratio \( \varepsilon /D \) of projections on the surface to the diameter of the pipe. Values of \( \varepsilon \) are as follows; glass and plastic pipe essentially have \( \varepsilon = 0 \).

\[ \begin{array}{c|c|c}
\text{pipe material} & \varepsilon (\text{ft}) & \varepsilon (\text{mm}) \\
\hline
\text{Riveted steel} & 0.003-0.03 & 0.9-9.0 \\
\text{Concrete} & 0.001-0.01 & 0.3-3.0 \\
\text{Wood stave} & 0.0006-0.003 & 0.18-0.9 \\
\text{Cast iron} & 0.00085 & 0.25 \\
\text{Galvanized iron} & 0.0005 & 0.15 \\
\text{Asphalted cast iron} & 0.0004 & 0.12 \\
\text{Commercial steel or wrought iron} & 0.00015 & 0.046 \\
\text{Drawn tubing} & 0.000005 & 0.0015 \\
\end{array} \]


\[
\frac{1}{\sqrt{f}} = 1.14 - 0.869 \ln \left( \frac{\varepsilon}{D} + \frac{9.38}{\text{Re}} \sqrt{1 + \frac{\varepsilon}{D} \frac{g}{g_e}} \right), \text{ } \text{Re} > 2100. \]

Other equations equivalent to this one but explicit in \( f \) have been devised. A literature review and comparison with more recent experimental data are made by Olujic [Chem. Eng., 91–94, (14 Dec 1981)]. Two of the simpler but adequate equations are

\[ f = 1.6364 \left( \ln \left( \frac{0.135 \varepsilon}{D} \frac{8}{\text{Re}} \right) \right)^{-2} \]


\[ f = \left( -0.8686 \ln \left( \frac{\varepsilon}{3.7 D} - 2.1802 \ln \left( \frac{\varepsilon}{3.7 D} + 14.5 \frac{g}{g_e} \right) \right) \right)^{-2} \]

[Schacham, Ind. Eng. Chem. Fundam. 19(5), 228 (1980)]. These
### TABLE 6.2. Typical Velocities and Pressure Drops in Pipelines

<table>
<thead>
<tr>
<th>Liquids (psi/100 ft)</th>
<th><strong>Liquids within 50°F of Bubble Point</strong></th>
<th><strong>Light Oils and Water</strong></th>
<th><strong>Viscous Oils</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pump suction</strong></td>
<td>0.15</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Pump discharge</strong></td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>(or 5–7 fps)</td>
<td>(or 5–7 fps)</td>
<td>(or 3–4 fps)</td>
</tr>
<tr>
<td><strong>Gravity flow to or from tankage, maximum</strong></td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Thermosyphon reboiler inlet and outlet</strong></td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Gases (psi/100 ft)

<table>
<thead>
<tr>
<th>Pressure (psig)</th>
<th>0–300 ft</th>
<th>Equivalent Length</th>
<th>300–600 ft</th>
<th>Equivalent Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>−13.7 (28 in. Vac)</td>
<td>0.06</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−12.2 (25 in. Vac)</td>
<td>0.10</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−7.5 (15 in. Vac)</td>
<td>0.15</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.25</td>
<td>0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.35</td>
<td>0.18</td>
<td></td>
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<tr>
<td>100</td>
<td>0.50</td>
<td>0.25</td>
<td></td>
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<tr>
<td>150</td>
<td>0.60</td>
<td>0.30</td>
<td></td>
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<tr>
<td>200</td>
<td>0.70</td>
<td>0.35</td>
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<tr>
<td>500</td>
<td>2.00</td>
<td>1.00</td>
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</tr>
</tbody>
</table>

**Steam (psi/100 ft Maximum ft/min)**

<table>
<thead>
<tr>
<th>Steam psi/100 ft</th>
<th>Maximum ft/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 50 psig</td>
<td>0.4</td>
</tr>
<tr>
<td>Over 50 psig</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Steam Condensate**

To traps, 0.2 psi/100 ft. From bucket traps, size on the basis of 2–3 times normal flow, according to pressure drop available. From continuous drainers, size on basis of design flow for 2.0 psi/100 ft.

**Control Valves**

Require a pressure drop of at least 10 psi for good control, but values as low as 5 psi may be used with some loss in control quality.

#### Particular Equipment Lines (ft/sec)

<table>
<thead>
<tr>
<th>Equipment Line</th>
<th>3–7</th>
<th>35–45</th>
<th>25–100</th>
<th>35–75</th>
<th>75–200</th>
<th>100–250</th>
<th>120–200</th>
<th>150–350</th>
<th>0.5(v_s)</th>
<th>(v_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reboiler, downcomer (liquid)</td>
<td></td>
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<tr>
<td>Reboiler, riser (liquid and vapor)</td>
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<tr>
<td>Overhead condenser</td>
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<tr>
<td>Two-phase flow</td>
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<td></td>
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<tr>
<td>Compressor, suction</td>
<td></td>
<td></td>
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<tr>
<td>Compressor, discharge</td>
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<tr>
<td>Inlet, steam turbine</td>
<td></td>
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<tr>
<td>Inlet, gas turbine</td>
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<tr>
<td>Relief valve, discharge</td>
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</tbody>
</table>

\(v_s\) is sonic velocity.

three equations agree with each other within 1% or so. The Colebrook equation predicts values 1–3% higher than some more recent measurements of Murin (1948), cited by Olujic (Chemical Engineering, 91–93, Dec. 14, 1981).

For orientation purposes, the pressure drop in steel pipes may be found by the rapid method of Table 6.3, which is applicable to highly turbulent flow for which the friction factor is given by von Karman’s equation:

\[
f = 1.3251\left[\ln(D/e) + 1.3123\right]^{-2}.
\]

Under some conditions it is necessary to employ Eq. (6.18) in differential form. In terms of mass flow rate,

\[
dP = \frac{8\pi^2f}{g\pi^2D^3}dL.
\]

Example 6.4 is of a case in which the density and viscosity vary along the length of the line, and consequently the Reynolds number and the friction factor also vary.

**FITTINGS AND VALVES**

Friction due to fittings, valves and other disturbances of flow in pipe lines is accounted for by the concepts of either their equivalent lengths of pipe or multiples of the velocity head. Accordingly, the pressure drop equation assumes either of the forms

\[
\Delta P = f(L + \sum L_i)\rho u^2/2gD,
\]

\[
\Delta P = f(L/D) + \sum K_i\rho u^2/2gD.
\]

Values of equivalent lengths \(L_i\) and coefficients \(K_i\) are given in Tables 6.4 and 6.5. Another well-documented table of \(K_i\) is in the Chemical Engineering Handbook (McGraw-Hill, New York, 1984 p. 5.38).

Comparing the two kinds of parameters,

\[
K_i = fL_i/D
\]

so that one or the other or both of these factors depend on the friction factor and consequently on the Reynolds number and possibly \(e\). Such a dependence was developed by Hooper [Chem. Eng., 96–100, (24 Aug. 1981)] in the equation

\[
K_i = K_{1i}/N_{Re} + K_2(1 + 1/D),
\]

where \(D\) is in inches and values of \(K_i\) and \(K_2\) are in Table 6.6. Hooper states that the results are applicable to both laminar and turbulent regions and for a wide range of pipe diameters. Example 6.5 compares the several systems of pipe fittings resistances. The \(K_i\) method usually is regarded as more accurate.

**ORIFICES**

In pipe lines, orifices are used primarily for measuring flow rates but sometimes as mixing devices. The volumetric flow rate through a thin plate orifice is

\[
Q = C_dA_0\left(\frac{2\Delta P/\rho}{1 - \beta^2}\right)^{1/2},
\]

\[
A_0 = \text{cross sectional area of the orifice,}
\]

\[
\beta = d/D, \text{ ratio of the diameters of orifice and pipe.}
\]

For corner taps the coefficient is given by

\[
C_d = 0.5959 + 0.0312\beta^2 - 0.184\beta^8
\]

\[+ (0.0029\beta^2)(10^6/Re_D)^{0.75}\]

(International Organization for Standards Report DIS 5167, Geneva, 1976). Similar equations are given for other kinds of orifice taps and for nozzles and Venturi meters.)
### TABLE 6.3. Approximate Computation of Pressure Drop of Liquids and Gases in Highly Turbulent Flow in Steel Pipes

<table>
<thead>
<tr>
<th>Nominal Pipe Size In.</th>
<th>Schedule Number</th>
<th>Value of ( C_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4</td>
<td>40 S</td>
<td>93,500</td>
</tr>
<tr>
<td>80 x</td>
<td>186,100</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>4,300,000</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>11,180,000</td>
<td></td>
</tr>
<tr>
<td>3/4</td>
<td>40 S</td>
<td>21,200</td>
</tr>
<tr>
<td>80 x</td>
<td>32,900</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>627,000</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>114,100</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>40 S</td>
<td>169</td>
</tr>
<tr>
<td>80 x</td>
<td>236</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>488</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>899</td>
<td></td>
</tr>
<tr>
<td>3/2</td>
<td>40 S</td>
<td>66.7</td>
</tr>
<tr>
<td>80 x</td>
<td>91.8</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>146.3</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>380.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>40 S</td>
<td>21.4</td>
</tr>
<tr>
<td>80 x</td>
<td>28.3</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>48.3</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>96.6</td>
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<tr>
<td>3/4</td>
<td>40 S</td>
<td>10.0</td>
</tr>
<tr>
<td>80 x</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>38.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>40 S</td>
<td>5.17</td>
</tr>
<tr>
<td>80 x</td>
<td>6.25</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>9.84</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>11.80</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>18.59</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>40 S</td>
<td>1.59</td>
</tr>
<tr>
<td>80 x</td>
<td>2.04</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>3.50</td>
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</tr>
<tr>
<td>6</td>
<td>40 S</td>
<td>0.610</td>
</tr>
<tr>
<td>80 x</td>
<td>0.758</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>1.015</td>
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</tr>
<tr>
<td>... xx</td>
<td>1.376</td>
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</tr>
<tr>
<td>8</td>
<td>40 S</td>
<td>0.133</td>
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<tr>
<td>80 x</td>
<td>0.185</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>0.289</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>0.317</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>40 S</td>
<td>0.0197</td>
</tr>
<tr>
<td>80 x</td>
<td>0.0596</td>
<td></td>
</tr>
<tr>
<td>... xx</td>
<td>0.0661</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>20 S</td>
<td>0.0157</td>
</tr>
<tr>
<td>30 S</td>
<td>0.0168</td>
<td></td>
</tr>
<tr>
<td>40 S</td>
<td>0.0175</td>
<td></td>
</tr>
<tr>
<td>60 S</td>
<td>0.0193</td>
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</tr>
<tr>
<td>14</td>
<td>10 S</td>
<td>0.00949</td>
</tr>
<tr>
<td>20 S</td>
<td>0.01046</td>
<td></td>
</tr>
<tr>
<td>30 S</td>
<td>0.01096</td>
<td></td>
</tr>
<tr>
<td>40 S</td>
<td>0.01156</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>10 S</td>
<td>0.00463</td>
</tr>
<tr>
<td>20 S</td>
<td>0.00421</td>
<td></td>
</tr>
<tr>
<td>30 S</td>
<td>0.00504</td>
<td></td>
</tr>
<tr>
<td>40 S</td>
<td>0.00549</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>10 S</td>
<td>0.00247</td>
</tr>
<tr>
<td>20 S</td>
<td>0.00266</td>
<td></td>
</tr>
<tr>
<td>30 S</td>
<td>0.00276</td>
<td></td>
</tr>
<tr>
<td>40 S</td>
<td>0.00298</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>10 S</td>
<td>0.00141</td>
</tr>
<tr>
<td>20 S</td>
<td>0.00150</td>
<td></td>
</tr>
<tr>
<td>30 S</td>
<td>0.00161</td>
<td></td>
</tr>
<tr>
<td>40 S</td>
<td>0.00189</td>
<td></td>
</tr>
<tr>
<td>10 S</td>
<td>0.000534</td>
<td></td>
</tr>
<tr>
<td>20 S</td>
<td>0.000565</td>
<td></td>
</tr>
<tr>
<td>30 S</td>
<td>0.000597</td>
<td></td>
</tr>
<tr>
<td>40 S</td>
<td>0.000614</td>
<td></td>
</tr>
<tr>
<td>60 S</td>
<td>0.000651</td>
<td></td>
</tr>
<tr>
<td>10 S</td>
<td>0.000741</td>
<td></td>
</tr>
<tr>
<td>20 S</td>
<td>0.000835</td>
<td></td>
</tr>
<tr>
<td>30 S</td>
<td>0.000972</td>
<td></td>
</tr>
<tr>
<td>40 S</td>
<td>0.001119</td>
<td></td>
</tr>
<tr>
<td>10 S</td>
<td>0.001478</td>
<td></td>
</tr>
</tbody>
</table>

\[ \Delta P_f = C_1 C_2 / \rho \text{ psi/100 ft}, \text{ with } \rho \text{ in lb/ft}^3. \]

(From Crane Co. Flow of Fluids through Fittings, Valves and Pipes, Crane Co., New York, 1982.)
**Example 6.4**

**Pressure Drop in Nonisothermal Liquid Flow**

Oil is pumped at the rate of 6000 lb/hr through a reactor made of commercial steel pipe 1.278 in. ID and 2000 ft long. The inlet condition is 400°F and 750 psia. The temperature of the outlet is 930°F and the pressure is to be found. The temperature varies with the distance, L ft, along the reactor according to the equation

\[ T = 1500 - 1100 \exp(-0.0003287L) \] (°F)

The viscosity and density vary with temperature according to the equations

\[ \mu = \exp\left(\frac{7445.3}{T + 459.6} - 6.1076\right), \text{ cP,} \]

\[ \rho = 0.936 - 0.00036T, \text{ g/mL.} \]

Round’s equation applies for the friction factor:

\[ \frac{N_{Re}}{\pi D \mu} = \frac{4(6000)}{\pi (1.278/12)^{2.42}} = \frac{29.641}{\mu} \]

\[ \varepsilon/D = \frac{0.00015(12)}{1.278} = 0.00141, \]

The differential pressure is given by

\[ -dP = \frac{8 \mu^2}{g \pi^2 \rho D^2} f dL = \frac{8(6000/3600)^2}{32.2 \pi^2 \rho (1.278/12)^2} \frac{351}{44} f dL, \text{ psi,} \]

\[ P = 750 - \int_0^L \frac{L}{f} dL = 750 - \int_0^L \frac{L}{f} dL. \]

The pressure profile is found by integration with the trapezoidal rule over 200 ft increments. The computer program and the printout are shown. The outlet pressure is 700.1 psia.

For comparison, taking an average temperature of 665°F,

\[ \mu = 1.670, \quad \rho = 0.697 \]

\[ N_{Re} = 17,700, \quad f = 0.00291, \]

\[ P_{out} = 702.5. \]
6.4. PIPELINE NETWORKS

A system for distribution of fluids such as cooling water in a process plant consists of many interconnecting pipes in series, parallel, or branches. For purposes of analysis, a point at which several lines meet is called a node and each is assigned a number as on the figure of Example 6.6. A flow rate from node $i$ to node $j$ is designated as $Q_{ij}$; the same subscript notation is used for other characteristics of the line such as $f$, $L$, $D$, and $N_{ij}$.

Three principles are applicable to establishing flow rates, pressures, and dimensions throughout the network:

1. Each node $i$ is characterized by a unique pressure $P_i$.
2. A material balance is preserved at each node: total flow in equals total flow out, or net flow equals zero.
3. The friction equation $P_i - P_j = (8/\rho g \pi^2) f_i L_i Q_{ij}^2 / D_i^5$ applies to the line connecting node $i$ with $j$.

In the usual network problem, the terminal pressures, line lengths, and line diameters are specified and the flow rates throughout are required to be found. The solution can be generalized, however, to determine other unknown quantities equal in number to the number of independent friction equations that describe the network. The procedure is illustrated with the network of Example 6.6.

The three lines in parallel between nodes 2 and 5 have the same pressure drop $P_2 - P_5$. In series lines such as 37 and 76 the flow rate is the same and a single equation represents friction in the series:

$$P_3 - P_6 = kQ_3^2 (f_{37} L_{37} / D_{37}^5 + f_{67} L_{67} / D_{67}^5).$$

The number of flow rates involved is the same as the number of lines in the network, which is 9, plus the number of supply and destination lines, which is 5, for a total of 14. The number of material balances equals the number of nodes plus one for the overall balance, making a total of 7.

The solution of the problem requires 14 - 7 = 7 more relations to be established. These are any set of 7 friction equations that involve the pressures at all the nodes. The material balances and pressure drop equations for this example are tabulated.

From Eqs. (4)–(10) of Example 6.6, any combination of seven quantities $Q_i$ and/or $L_i$ and/or $D_i$ can be found. Assuming that the $Q_i$ are to be found, estimates of all seven are made to start, and the corresponding Reynolds numbers and friction factors are found from Eqs. (2) and (3). Improved values of the $Q_i$ then are found

### Table 6.4. Equivalent Lengths of Pipe Fittings

<table>
<thead>
<tr>
<th>Pipe size, in.</th>
<th>Standard ell</th>
<th>Medium ell</th>
<th>Long-radius ell</th>
<th>45-deg ell</th>
<th>Tee</th>
<th>Gate valve, open</th>
<th>Globe valve, open</th>
<th>Swing check, open</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.7</td>
<td>2.3</td>
<td>1.7</td>
<td>1.3</td>
<td>5.8</td>
<td>0.6</td>
<td>27</td>
<td>6.7</td>
</tr>
<tr>
<td>2</td>
<td>5.5</td>
<td>4.6</td>
<td>3.5</td>
<td>2.5</td>
<td>11.0</td>
<td>1.2</td>
<td>57</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>8.1</td>
<td>6.8</td>
<td>5.1</td>
<td>3.8</td>
<td>17.0</td>
<td>1.7</td>
<td>85</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>11.0</td>
<td>9.1</td>
<td>7.0</td>
<td>5.0</td>
<td>22</td>
<td>2.3</td>
<td>110</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>14.0</td>
<td>12.0</td>
<td>8.9</td>
<td>6.1</td>
<td>27</td>
<td>2.0</td>
<td>140</td>
<td>33</td>
</tr>
<tr>
<td>6</td>
<td>16.0</td>
<td>14.0</td>
<td>11.0</td>
<td>7.7</td>
<td>33</td>
<td>3.5</td>
<td>160</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
<td>18.0</td>
<td>14.0</td>
<td>10.0</td>
<td>43</td>
<td>4.5</td>
<td>220</td>
<td>53</td>
</tr>
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<td>10</td>
<td>26</td>
<td>22</td>
<td>17.0</td>
<td>13.0</td>
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<td>5.7</td>
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<td>67</td>
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<td>26</td>
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<td>31</td>
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<td>53</td>
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<td>14.0</td>
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<tr>
<td>35</td>
<td>94</td>
<td>79</td>
<td>60</td>
<td>43</td>
<td>200</td>
<td>20.0</td>
<td>1,000</td>
<td>240</td>
</tr>
</tbody>
</table>

*Length of straight pipe (ft) giving equivalent resistance. (Hicks and Edwards, Pump Application Engineering, McGraw-Hill, New York, 1971)*.
TABLE 6.5. Velocity Head Factors of Pipe Fittings

\[ h = \frac{K u^2}{2 g} \text{ ft of fluid.} \]

(Hydraulic Institute, Cleveland, OH, 1957).
TABLE 6.6. Velocity Head Factors of Pipe Fittings

<table>
<thead>
<tr>
<th>Fitting type</th>
<th>$K_1$</th>
<th>$K_{oo}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard (R/D = 1), screwed</td>
<td>800</td>
<td>0.40</td>
</tr>
<tr>
<td>Standard (R/D = 1), flanged/welded</td>
<td>800</td>
<td>0.25</td>
</tr>
<tr>
<td>Long-radius (R/D = 1.5), all types</td>
<td>800</td>
<td>0.20</td>
</tr>
<tr>
<td>90\degree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mitered</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Weld (90\degree angle)</td>
<td>1,000</td>
<td>1.15</td>
</tr>
<tr>
<td>2 Weld (45\degree angles)</td>
<td>800</td>
<td>0.35</td>
</tr>
<tr>
<td>3 Weld (30\degree angles)</td>
<td>800</td>
<td>0.30</td>
</tr>
<tr>
<td>4 Weld (22\degree angles)</td>
<td>800</td>
<td>0.27</td>
</tr>
<tr>
<td>5 Weld (18\degree angles)</td>
<td>800</td>
<td>0.25</td>
</tr>
<tr>
<td>45\degree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard (R/D = 1), all types</td>
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<td>0.20</td>
</tr>
<tr>
<td>Long-radius (R/D = 1.5), all types</td>
<td>500</td>
<td>0.15</td>
</tr>
<tr>
<td>Mitered, 1 weld, 45\degree angle</td>
<td>500</td>
<td>0.25</td>
</tr>
<tr>
<td>Mitered, 2 weld, 22\degree angles</td>
<td>500</td>
<td>0.15</td>
</tr>
<tr>
<td>180\degree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard (R/D = 1), screwed</td>
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<td>0.60</td>
</tr>
<tr>
<td>Standard (R/D = 1), flanged/welded</td>
<td>1,000</td>
<td>0.35</td>
</tr>
<tr>
<td>Long-radius (R/D = 1.5), all types</td>
<td>1,000</td>
<td>0.30</td>
</tr>
<tr>
<td>Run-through tee</td>
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<td></td>
</tr>
<tr>
<td>Screwed</td>
<td>200</td>
<td>0.10</td>
</tr>
<tr>
<td>Full line size, $\beta = 1.0$</td>
<td>300</td>
<td>0.10</td>
</tr>
<tr>
<td>Flanged or welded</td>
<td>150</td>
<td>0.50</td>
</tr>
<tr>
<td>Stub-in-type branch</td>
<td>100</td>
<td>0.00</td>
</tr>
<tr>
<td>Gate,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full line size, $\beta = 1.0$</td>
<td>300</td>
<td>0.10</td>
</tr>
<tr>
<td>Flanged or welded</td>
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<td>0.50</td>
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<tr>
<td>Stub-in-type branch</td>
<td>100</td>
<td>0.00</td>
</tr>
<tr>
<td>Valve</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Globe, standard</td>
<td>1,500</td>
<td>4.00</td>
</tr>
<tr>
<td>Globe, angle or Y-type</td>
<td>1,000</td>
<td>2.00</td>
</tr>
<tr>
<td>Diaphragm, dam type</td>
<td>1,000</td>
<td>2.00</td>
</tr>
<tr>
<td>Butterfly</td>
<td>800</td>
<td>0.25</td>
</tr>
<tr>
<td>Check</td>
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<td></td>
</tr>
<tr>
<td>Lift</td>
<td>2,000</td>
<td>10.00</td>
</tr>
<tr>
<td>Swing</td>
<td>1,500</td>
<td>1.50</td>
</tr>
<tr>
<td>Titting-disk</td>
<td>1,000</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: Use $R/D = 1.5$ for $R/D = 5$ pipe bends, 45\degree to 180\degree. Use appropriate tee values for flow through tees.

\* Inlet flush, $K = 160/N_{in} + 0.6$. Inlet, intruding, $K = 160/N_{in} = 1.0$. Exit, $K = 1.0$, $K = K_{In}/N_{in} + K_{In}/1 + 1/D$, with D in inches. (Hooper, Chem. Eng. 96–100 (24 Aug. 1981)).

6.5. OPTIMUM PIPE DIAMETER

In a chemical plant the capital investment in process piping is in the range of 25–40% of the total plant investment, and the power consumption for pumping, which depends on the line size, is a substantial fraction of the total cost of utilities. Accordingly, economic optimization of pipe size is a necessary aspect of plant design. As the diameter of a line increases, its cost goes up but is accompanied by decreases in consumption of utilities and costs of pumps and drivers because of reduced friction. Somewhere there is an optimum balance between operating cost and annual capital cost.

For small capacities and short lines, near optimum line sizes may be obtained on the basis of typical velocities or pressure drops such as those of Table 6.2. When large capacities are involved and lines are long and expensive materials of construction are needed, the selection of line diameters may need to be subjected to complete economic analysis. Still another kind of factor may need to be taken into account with highly viscous materials: the possibility that heating the fluid may pay off by reducing the viscosity and consequently the power requirement.

Adequate information must be available for installed costs of piping and pumping equipment. Although suppliers quotations are desirable, published correlations may be adequate. Some data and references to other published sources are given in Chapter 20. A simplification in locating the optimum usually is permissible by ignoring the costs of pumps and drivers since they are essentially insensitive to pipe diameter near the optimum value. This fact is clear in Example 6.8 for instance and in the examples worked out by Happel and Jordan (Chemical Process Economics, Dekker, New York, 1975).

Two shortcut rules have been derived by Peters and Timmerhaus (1980; listed in Chapter 1 References) for optimum diameters of steel pipes of 1-in. size or greater, for turbulent and laminar flow:

$$D = 3.900.45\rho^{0.13}, \text{ turbulent flow,}$$
$$D = 3.0Q^{0.36}\mu^{0.18}, \text{ laminar flow.}$$

$D$ is in inches, $Q$ in cuft/sec, $\rho$ in lb/cuft, and $\mu$ in cP. The factors involved in the derivation are: power cost = $0.055/kWh$, friction loss due to fittings is 35% that of the straight length, annual fixed charges are 20% of installation cost, pump efficiency is 50%, and cost of 1-in. IPS schedule 40 pipe is $0.45/ft. Formulas that take additional factors into account also are developed in that book.

Other detailed studies of line optimization are made by Happel and Jordan (Chemical Process Economics, Dekker, New York, 1975) and by Skelland (1967). The latter works out a problem in simultaneous optimization of pipe diameter and pumping temperature in laminar flow.

Example 6.8 takes into account pump costs, alternate kinds of drivers, and alloy construction.

6.6. NON-NEWTONIAN LIQUIDS

Not all classes of fluids conform to the frictional behavior described in Section 6.3. This section will describe the commonly recognized types of liquids, from the point of view of flow behavior, and will summarize the data and techniques that are used for analyzing friction in such lines.

VISCOSITY BEHAVIOR

The distinction in question between different fluids is in their viscosity behavior, or relation between shear stress $\tau$ (force per unit area) and the rate of deformation expressed as a lateral velocity...
Example 6.5
Comparison of Pressure Drops in a Line with Several Sets of Fittings Resistances
The flow considered is in a 12-inch steel line at a Reynolds number of 6000. With \(\varepsilon = 0.00015\), Round’s equation gives \(f = 0.0353\). The line composition and values of fittings resistances are:

Table 6.4, \(\frac{\Delta P}{(\rho u^2/2g)}D\) (1738) = 61.3,

Table 6.5, \(\frac{\Delta P}{(\rho u^2/2g)}D = \sum K_i\)

\[= \frac{0.0353(1000)}{1} + 9.00 = 44.3,\]

Table 6.6, \(\frac{\Delta P}{(\rho u^2/2g)}D = 35.3 + 8.64 = 43.9.\)

The value \(K = 0.05\) for gate valve from Table 6.5 appears to be low: Chemical Engineering Handbook, for example, gives 0.17, more nearly in line with that from Table 6.6. The equivalent length method of Table 6.4 gives high pressure drops; although convenient, it is not widely used.

Example 6.6
A Network of Pipelines in Series, Parallel, and Branches: the Sketch, Material Balances, and Pressure Drop Equations
Pressure drop:

\[\Delta P = \left(8\rho g\pi^2\right)f_h L_2 Q_p^2 / D_2 = f_h L_2 Q_p^2 / D_2.\]  (1)

Reynolds number:

\[\left(N_{Re}\right)_{rl} = 4Q_p D / \pi D_2 \mu.\]  (2)

Friction factor:

\[f_h = 1.6364/\left[\ln\left(\varepsilon / D_2 + 6.5/(N_{Re} \rho)\right)\right]^2.\]  (3)

Pressure drops in key lines:

\[\Delta P_{21} = P_1 - P_2 - \kappa f_2 L_2 Q_{21}^2 / D_2 = 0,\]  (4)

\[\Delta P_{23} = P_2 - P_3 - 2\kappa f_2 L_2 Q_{23}^2 / D_2 = 0,\]  (5)

\[\Delta P_{25} = P_2 - P_5 - \kappa f_2 L_2 Q_{25}^2 / D_2 = 0,\]  (6)

\[= f_h L_2 Q_{25}^2 / (D_2)^5 \]  (7)

\[= f_h L_2 Q_{25}^2 / (D_2)^5,\]  (8)

\[\Delta P_{42} = P_4 - P_5 - \kappa f_4 L_2 Q_{42}^2 / D_2 = 0,\]  (9)

\[\Delta P_{45} = P_3 - P_5 - \kappa f_4 L_2 Q_{45}^2 / D_2 = 0\]  (10)

Material balance at node:

\[Q_{o1} = Q_{12} - Q_{14} = 0\]  (11)

\[Q_{12} - Q_{23} - Q_{25}^{(1)} - Q_{25}^{(2)} = 0\]  (12)

\[Q_{32} - Q_{23} - Q_{26} = 0\]  (13)

\[Q_{14} - Q_{20} - Q_{25}^{(1)} = 0\]  (14)

\[Q_{24} + Q_{25}^{(1)} + Q_{25}^{(2)} + Q_{25}^{(3)} - Q_{20} - Q_{26} = 0\]  (15)

\[Q_{26} - Q_{20} - Q_{26} = 0\]  (16)

Overall \[Q_{o1} + Q_{o3} - Q_{o0} - Q_{o1} - Q_{o0} = 0\]  (17)

Example 6.7
Flow of Oil in a Branched Pipeline
The pipeline handles an oil with sp gr = 0.92 and kinematic viscosity of 5 centistokes(cS) at a total rate of 12,000 cuft/hr. All three pumps have the same output pressure. At point 5 the elevation is 100 ft and the pressure is 2 atm gage. Elevations at the other points are zero. Line dimensions are tabulated following. The flow rates in each of the lines and the total power requirement will be found.

\[Q_1 + Q_2 + Q_3 = Q_4 = 12,000/3600 = 3.333\ cfs\]  (1)

\[N_{Re} = \frac{4Q_4}{\pi D^2} = \frac{4Q_4}{\pi D(5/92,900)} = 23,657Q_4 / D\]

\[= \left[\begin{array}{c} 59,142Q_1 \\ 47,313Q_2 \\ 78,556Q_3 \\ 31,542Q_4 \end{array}\right].\]  (2)
EXAMPLE 6.7—(continued)

\[ e = 0.00015 \text{ ft}, \]
\[ h_f = \frac{8fLQ^2}{D^3} = 0.0251fLQ^2/D^4 \text{ ft}, \]  \hspace{1cm} (3)
\[ h_f = h_{j2} = h_{j3}, \]
\[ \frac{Q_1^2}{D_1^2} = \frac{Q_2^2}{D_2^2} = \frac{Q_3^2}{D_3^2} \]  \hspace{1cm} (4)
\[ \frac{Q_2}{Q_1} = \left[ \frac{f_1}{f_2} \right]^4 \]  \hspace{1cm} (5)
\[ Q_2 = \frac{Q_1^2}{(\frac{f_2}{f_1})^{1/2}} = \frac{(1.2352)^4f_2}{Q_1^2} \]  \hspace{1cm} (6)
\[ Q_3 = \frac{Q_1^2}{(\frac{f_3}{f_1})^{1/2}} = \frac{(0.3977)^4f_3}{Q_1^2} \]  \hspace{1cm} (7)
\[ Q_4 = \frac{1 + (1.2352)^2f_2}{(\frac{Q_2}{Q_1})^{1/2}} = \frac{0.3977f_3}{(\frac{Q_3}{Q_1})^{1/2}} = 0.00015 \text{ ft}, \]
\[ hf_{J4} = hf_{J2} = hf_3 = \frac{0.0251(0.02068)(1000)(1.2707)^2}{(0.4)^2} = 82.08 \text{ ft}. \]

Velocity head at discharge:
\[ \frac{w_2^2}{2g_c} = \frac{1}{2g_c} \left( \frac{Q_4}{\pi/4D^2} \right)^2 = 0.88 \text{ ft}. \]

Total head at pumps:
\[ h_p = \frac{2(2117)}{0.92(62.4)} + 100 \]
\[ + 0.88 + 82.08 + 88.65 \]
\[ = 345.36 \text{ ft}. \]

Power
\[ = \gamma Q h_p \]
\[ = 0.92(62.4)(10/3)345.36 \]
\[ = 66,088 \text{ ft lb/sec} \]
\[ 120.2 \text{ HP, 89.6 kW}. \]

For line 45,
\[ (N_{Re})_4 = 31542 (3.333) = 105,140, \]
\[ f_4 = 0.01881, \]
\[ (h_f)_{45} = \frac{0.0251(0.01881)(4000)(3.333)^2}{(0.75)^2} = 88.65 \text{ ft}. \]

Procedure:
1. As a first trial assume \( f_1 = f_2 = f_3 \), and find \( Q_1 = 1.266 \) from Eq. (8).
2. Find \( Q_2 \) and \( Q_3 \) from Eqs. (6) and (7).
3. With these values of the \( Q_i \), find improved values of the \( f_i \) and hence improved values of \( Q_2 \) and \( Q_3 \) from Eqs. (6) and (7).
4. Check how closely \( Q_1 + Q_2 + Q_3 = 3.333 \) = 0.
5. If check is not close enough, adjust the value of \( Q_1 \) and repeat the calculations.

The two trials shown following prove to be adequate.

\[ Q_1 \quad Q_2 \quad Q_3 \quad \text{Summary} \]
\[ 1.2660 \quad 1.5757 \quad 0.4739 \quad 0.0023 \quad 0.02069 \]
\[ 1.2707 \quad 1.5564 \quad 0.0573 \quad 0.0001 \quad 0.02068 \]

Summary:

<table>
<thead>
<tr>
<th>Line</th>
<th>( N_{Re} )</th>
<th>( f )</th>
<th>( Q )</th>
<th>( h_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>75.152</td>
<td>0.02068</td>
<td>1.2707</td>
<td>82.08</td>
</tr>
<tr>
<td>24</td>
<td>60.121</td>
<td>0.02106</td>
<td>1.5564</td>
<td>82.08</td>
</tr>
<tr>
<td>34</td>
<td>99.821</td>
<td>0.02053</td>
<td>0.5073</td>
<td>82.08</td>
</tr>
<tr>
<td>45</td>
<td>105.140</td>
<td>0.01881</td>
<td>3.3333</td>
<td>88.65</td>
</tr>
</tbody>
</table>

\[ \text{Characteristics of the alternate pump drives are:} \]
\[ \text{a. Turbines are 3600 rpm, exhaust pressure is 0.75 bar, inlet pressure is 20 bar, turbine efficiency is 45\%. Value of the high pressure steam is $5.25/1000 lbs; that of the exhaust is $0.75/1000 lbs.} \]
\[ \text{b. Motors have efficiency of 90\%, cost of electricity is $0.065/kWh.} \]
\[ \text{Cost data are:} \]
\[ \text{1. Installed cost of pipe is 7.5D$/ft and that of valves is 600D^{0.7}} \text{ each, where D is the nominal pipe size in inches.} \]

3. All prices are as of mid-1975. Escalation to the end of 1984 requires a factor of 1.8. However, the location of the optimum will be approximately independent of the escalation if it is assumed that equipment and utility prices escalate approximately uniformly; so the analysis is made in terms of the 1975 prices. Annual capital cost is 50% of the installed price/year.

The summary shows that a 6-in. line is optimum with motor drive, and an 8-in. line with turbine drive. Both optima are insensitive to line sizes in the range of 6–10 in.

\[ Q = \frac{1000}{(7.48)(60)} = 2.2282 \text{ cfs}, \quad 227.2 \text{ m}^3/\text{hr}, \]

\[ N_{\text{Re}} = \frac{4Qp}{\pi D \mu} = \frac{4(2.2282)(0.81)(62.4)}{71.128} \]

\[ f = 1.6364 \left[ \frac{0.135(0.0015)}{71.128} + \frac{6.5D}{71.128} \right] \]

\[ h_p = \frac{120(144)}{0.81(62.4)} \frac{8/LQ^2}{g} \text{ in}^2/\text{ft}^3 = 341.88 + 124.98f/D^3 \text{ ft.} \]

Motor power:

\[ P_m = \frac{Qp}{\eta p \eta m} = \frac{2.2282(50.54)}{550(0.71(0.90))} h_p = 0.3204h_p, \quad \text{HP} \]

Turbine power:

\[ P_t = \frac{2.2282(50.54)}{550(0.71)} h_p = 0.2883h_p, \quad \text{HP}. \]

Gradient, \( \gamma = du/dx \). The concept is represented on Figure 6.2(a): one of the planes is subjected to a shear stress and is translated parallel to a fixed plane at a constant velocity but a velocity gradient is developed between the planes. The relation between the variables may be written

\[ \tau = F/A = \mu(du/dx) = \mu \gamma, \quad (6.34) \]

where, by definition, \( \mu \) is the viscosity. In the simplest case, the viscosity is constant, and the fluid is called Newtonian. In the other cases, more complex relations between \( \tau \) and \( \gamma \) involving more than one constant are needed, and dependence on time also may be present. Classifications of non-Newtonian fluids are made according to the relation between \( \tau \) and \( \gamma \) by formula or shape of plot, or according to the mechanism of the resistance of the fluid to deformation.

The concept of an apparent viscosity

\[ \mu_a = \tau/\gamma \quad (6.35) \]

is useful. In the Newtonian case it is constant, but in general it can be a function of \( \gamma \), \( \gamma \), and time \( \theta \).

Non-Newtonian behavior occurs in solutions or melts of polymers and in suspensions of solids in liquids. Some \( \tau-\gamma \) plots are shown in Figure 6.2, and the main classes are described following.

1. **Pseudoplastic liquids** have a \( \tau-\gamma \) plot that is concave downward. The simplest mathematical representation of such relations is a power law

\[ \tau = K\gamma^n, \quad n < 1 \quad (6.36) \]

with \( n < 1 \). This equation has two constants; others with many more than two constants also have been proposed. The apparent viscosity is

\[ \mu_a = \tau/\gamma = K/\gamma^{1-n}. \quad (6.37) \]

Since \( n \) is less than unity, the apparent viscosity decreases with the deformation rate. Examples of such materials are some polymeric solutions or melts such as rubbers, cellulose acetate and napalm; suspensions such as paints, mayonnaise, paper pulp, or detergent slurries; and dilute suspensions of inert solids. Pseudoplastic properties of wallpaper paste account for good spreading and adhesion, and those of printing inks prevent their running at low speeds yet allow them to spread easily in high speed machines.

2. **Dilatant liquids** have rheological behavior essentially
Figure 6.2. Relations between shear stress, deformation rate, and viscosity of several classes of fluids. (a) Distribution of velocities of a fluid between two layers of areas $A$ which are moving relatively to each other at a distance $x$ under influence of a force $F$. In the simplest case, $F/A = \mu (du/dx)$ with $\mu$ constant. (b) Linear plot of shear stress against deformation. (c) Logarithmic plot of shear stress against deformation rate. (d) Viscosity as a function of shear stress. (e) Time-dependent viscosity behavior of a rheopectic fluid (thixotropic behavior is shown by the dashed line). (f) Hysteresis loops of time-dependent fluids (arrows show the chronology of imposed shear stress).
opposite those of pseudopastics insofar as viscosity behavior is concerned. The \( \tau - \dot{\gamma} \) plots are concave upward and the power law applies

\[
\tau = K\dot{\gamma}^n, \quad n > 1,
\]

but with \( n \) greater than unity; other mathematical relations also have been proposed. The apparent viscosity, \( \mu_a = K\dot{\gamma}^{n-1} \), increases with deformation rate. Examples of dilatant materials are pigment-vehicle suspensions such as paints and printing inks of high concentrations; starch, potassium silicate, and gum arabic in water; quicksand or beach sand in water. Dilatant properties of wet cement aggregates permit tamping operations in which small impulses produce more complete settling. Vinyl resin plastisols exhibit pseudoplastic behavior at low deformation rates and dilatant Bingham characteristics allow toothpaste to stay on the brush.

3. Bingham plastics require a finite amount of shear stress before deformation begins, then the deformation rate is linear. Mathematically,

\[
\tau = \tau_0 + \mu_p (du/dx) = \tau_0 + \mu_p \dot{\gamma},
\]

where \( \mu_p \) is called the coefficient of plastic viscosity. Examples of materials that approximate Bingham behavior are drilling muds; suspensions of chalk, grains, and thorium; and sewage sludge. Bingham characteristics allow toothpaste to stay on the brush.

4. Generalized Bingham or yield-power law fluids are represented by the equation

\[
\tau = \tau_0 + K\dot{\gamma}^n.
\]

Yield-dilatant (\( n > 1 \)) materials are rare but several cases of yield-pseudopastics exist. For instance, data from the literature of a 20% clay in water suspension are represented by the numbers \( \tau_0 = 7.3 \text{ dyn/cm}^2 \), \( K = 1.296 \text{ dyn/(sec)} \text{ cm}^2 \), and \( n = 0.483 \) (Govier and Aziz, 1972, p. 40). Solutions of 0.5–5.0% carboxypolymethylene also exhibit this kind of behavior, but at lower concentrations the yield stress is zero.

5. Rheopectic fluids have apparent viscosities that increase with time, particularly at high rates of shear as shown on Figure 6.3. Figure 6.2(f) indicates typical hysteresis effects for such materials. Some examples are suspensions of gypsum in water, bentonite sols, vanadium pentoxide sols, and the polyester of Figure 6.3.

6. Thixotropic fluids have a time-dependent rheological behavior in which the shear stress diminishes with time at a constant deformation rate, and exhibits hysteresis [Fig. 6.2(f)]. Among the substances that behave this way are some paints, ketchup, gelatine solutions, mayonnaise, margarine, mustard, honey, and shaving cream. Nondrip paints, for example, are thick in the can but thin on the brush. The time-effect in the case of the thixotropic crude of Figure 6.4(a) diminishes at high rates of deformation. For the same crude, Figure 6.4(b) represents the variation of pressure gradient in a pipeline with time and axial position; the gradient varies fivefold over a distance of about 2 miles after 200 min. A relatively simple relation involving five constants to represent thixotropic behavior is cited by Govier and Aziz (1972, p. 43):

\[
\dot{\chi}/D = a - (a + b\dot{\gamma})\lambda.
\]

The constants \( \mu_0, a, b, \) and \( \lambda \) and the structural parameter \( \lambda \) are obtained from rheological measurements in a straightforward manner.

7. Viscoelastic fluids have the ability of partially recovering their original states after stress is removed. Essentially all molten polymers are viscoelastic as are solutions of long chain molecules such as polyethylene oxide, polyacrylamides, sodium carboxymethylcellulose, and others. More homely examples are egg whites, dough, jello, and puddings, as well as bitumen and napalm. This property enables eggwhites to entrap air, molten polymers to form threads, and such fluids to climb up rotating shafts whereas purely viscous materials are depressed by the centrifugal force. Two concepts of deformability that normally are applied only to solids, but appear to have examples of gradation between solids and liquids, are those of shear modulus \( E \), which is

\[
E = \text{shear stress/deformation},
\]

and relaxation time \( \theta^* \), which is defined in the relation between the residual stress and the time after release of an imposed shear stress, namely,

\[
\tau = \tau_0 \exp(-\theta/\theta^*).
\]

A range of values of the shear modulus (in kgf/cm²) is

<table>
<thead>
<tr>
<th>Material</th>
<th>( E ) (kgf/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gelatine</td>
<td>( 4 \times 10^{-20} )</td>
</tr>
<tr>
<td>0.5% solution</td>
<td>( 4 \times 10^{-20} )</td>
</tr>
<tr>
<td>10% solution</td>
<td>( 5 \times 10^{-2} )</td>
</tr>
<tr>
<td>Raw rubber</td>
<td>( 1.7 \times 10^7 )</td>
</tr>
<tr>
<td>Lead</td>
<td>( 4.8 \times 10^6 )</td>
</tr>
<tr>
<td>Wood (oak)</td>
<td>( 8 \times 10^6 )</td>
</tr>
<tr>
<td>Steel</td>
<td>( 8 \times 10^6 )</td>
</tr>
</tbody>
</table>

**Figure 6.3.** Time-dependent rheological behavior of a rheopectic fluid, a 2000 molecular weight polyester [after Steg and Katz, J. Appl. Polym. Sci. 9, 3177 (1965)].
The sizing of pipelines for non-Newtonian liquids may be based on scaleup of tests made under the conditions at which the proposed line is to operate, without prior determination and correlation of rheological properties. A body of theory and some correlations are available for design with four mathematical models:

\[ \tau_w = K \gamma^n \]  
\[ \tau_w = \tau_y + \mu \dot{\gamma} \]  
\[ \tau_w = \tau_y + K' \gamma^n \]  
\[ \tau_w = K' \left( \frac{8V}{D} \right)^n \]

In the last model, the parameters may be somewhat dependent on the shear stress and deformation rate, and should be determined at magnitudes of those quantities near those to be applied in the plant.

The shear stress \( \tau_w \) at the wall is independent of the model and is derived from pressure drop measurements as

\[ \tau_w = \frac{D \Delta P}{4L}. \]

**Friction Factor.** In rheological literature the friction factor is defined as

\[ f = \frac{D \Delta P}{4L \rho V^2/2g_c} = \frac{\tau_w}{p V^2/2g_c}. \]

By analogy with the Newtonian relation, \( f = 16/Re \), the denominator of Eq. (6.52) is designated as a modified Reynolds number,

\[ Re_{MR} = D^n V^{2-n} \rho /g_c K' 8^{n-1}. \]

The subscript MR designates Metzner–Reed, who introduced this form.

In critical cases of substantial economic importance, it may be advisable to perform flow tests—\( Q \) against \( \Delta P \)—in lines of moderate size and to scale up the results to plant size, without necessarily trying to fit one of the accepted models. Among the effects that may not be accounted for by such models are time

**Scale Up.** The design of pipelines and other equipment for handling non-Newtonian fluids may be based on model equations with parameters obtained on the basis of measurements with viscometers or with pipelines of substantial diameter. The shapes of plots of \( \tau_w \) against \( \gamma \) or \( \dot{\gamma} \) or \( 8V/D \) may reveal the appropriate model.

Examples 6.9 and 6.10 are such analyses.
EXAMPLE 6.9
Analysis of Data Obtained in a Capillary Tube Viscometer

Data were obtained on a paper pulp with specific gravity 1.3, and are given as the first four columns of the table. Shear stress $\tau_w$ and deformation rate $\dot{\gamma}$ are derived by the equations applying to this kind of viscometer (Skelland, 1967, p. 31; Van Wazer et al., 1963, p. 197):

$$\tau_w = \frac{D\Delta P}{4L},$$
$$\dot{\gamma} = \frac{3n' + 1}{4n'} \left( \frac{8V}{D} \right),$$
$$n' = -\frac{d \ln(\tau_w)}{d \ln(8V/D)}$$

The plot of log $\tau_w$ against log $(8V/D)$ shows some scatter but is approximated by a straight line with equation

$$\tau_w = 1.329(8V/D)^{0.51}.$$  

Since

$$\dot{\gamma} = (2.53/2.08)(8V/D),$$

the relation between shear stress and deformation is given by the equation

$$\tau_w = 1.203\dot{\gamma}^{0.51}.$$  


EXAMPLE 6.10
Parameters of the Bingham Model from Measurements of Pressure Drops in a Line

Data of pressure drop in the flow of a 60% limestone slurry of density 1.607 g/ml were taken by Thomas [Ind. Eng. Chem. 55, 18-29 (1963)]. They were converted into data of wall shear stress $\tau_w = D\Delta P/4L$ against the shear rate $8V/D$ and are plotted on the figure for three line sizes.

The Buckingham equation for Bingham flow in the laminar region is

$$\frac{8V}{D} = \frac{\tau_w}{\mu_B} \left[ 1 - \frac{4}{3} \left( \frac{\tau_0}{\tau_w} \right)^{1/3} \right]^{4/3}$$

$$= \frac{1}{\mu_B} \left( \tau_w - \frac{4}{3} \tau_0 \right)$$

The second expression is obtained by neglecting the fourth-power term. The Bingham viscosity $\mu_B$ is the slope of the plot in the laminar region and is found from the terminal points as

$$\mu_B = \frac{(73-50)/(347-0)}{0.067 \text{ dyn sec/cm}^2}.$$

From the reduced Buckingham equation,

$$\tau_0 = 0.75\tau_w \text{ (at } 8V/D = 0),$$
$$\tau_0 = 37.5.$$  

Accordingly, the Bingham model is represented by

$$\tau_w = 37.5 + 0.067(8V/D), \text{ dyn/cm}^2$$

with time in seconds.

Transitions from laminar to turbulent flow may be identified off the plots:

$$D = 2.06 \text{ cm}, \quad 8V/D = 465, \quad V = 120 \text{ cm/sec}$$
$$4.04 \quad 215, \quad 109$$
$$7.75 \text{ (critical not reached).}$$

The transition points also can be estimated from Hanks' correlation [AICHE J. 9, 45, 306 (1963)] which involves these expressions:

$$x_c = \frac{(\tau_0/\tau_w)_c}{},$$
$$He = D^2\tau_0\rho/\mu_B^2,$$
$$x_c/(1 - x_c)^3 = He/16,800,$$
$$Re_{He} = (1 - \frac{2}{3}x_c - \frac{4}{5}x_c^2)He/x_c.$$  

The critical linear velocity finally is evaluated from the critical Reynolds number of the last equation with the following results:
dependence, pipe roughness, pipe fitting resistance, wall slippage, and viscoelastic behavior. Although some effort has been devoted to them, none of these particular effects has been well correlated. Viscoelasticity has been found to have little effect on friction in straight lines but does have a substantial effect on the resistance of pipe fittings. Pipe roughness often is accounted for by assuming that the relative effects of different roughness ratios \( \varepsilon/D \) are represented by the Colebrook equation (Eq. 6.20) for Newtonian fluids. Wall slippage due to trace amounts of some polymers in solution is an active field of research (Hoyt, 1972) and is not well predictable.

The scant literature on pipeline scaleup is reviewed by Heywood (1980). Some investigators have assumed a relation of the form

\[
\tau_w = \frac{DAP}{4L} = kV^a/D^b
\]

and determined the three constants \( K, a, \) and \( b \) from measurements on several diameters of pipe. The exponent \( a \) on the velocity appears to be independent of the diameter if the roughness ratio \( \varepsilon/D \) is held constant. The exponent \( b \) on the diameter has been found to range from 0.2 to 0.25. How much better this kind of analysis is than assuming that \( a = b \), as in Eq. (6.48), has not been established. If it can be assumed that the effect of differences in \( \varepsilon/D \) is small for the data of Examples 6.9 and 6.10, the measurements should plot as separate lines for each diameter, but such a distinction is not obvious on those plots in the laminar region, although it definitely is in the turbulent region of the limestone slurry data.

Observations of the performance of existing large lines, as in the case of Figure 6.4, clearly yields information of value in analyzing the effects of some changes in operating conditions or for the design of new lines for the same system.

**Laminar Flow.** Theoretically derived equations for volumetric flow rate and friction factor are included for several models in Table 6.7. Each model employs a specially defined Reynolds number, and the Bingham models also involve the Hedstrom number,

\[
He = \frac{\tau_w D^2}{\mu_p}.
\]

These dimensionless groups also appear in empirical correlations of the turbulent flow region. Although even in the approximate Eq. (9) of Table 6.7, group \( He \) appears to affect the friction factor, empirical correlations such as Figure 6.5(b) and the data analysis of Example 6.10 indicate that the friction factor is determined by the Reynolds number alone, in every case by an equation of the form,

\[
f = \frac{16}{Re}.
\]

**Transitional Flow.** Reynolds numbers and friction factors at which the flow changes from laminar to turbulent are indicated by the breaks in the plots of Figures 6.4(a) and (b). For Bingham models, data are shown directly on Figure 6.6. For power-law flow the correlation of Dodge and Metzner (1959) is shown in Figure 6.5(a) and is represented by the equation

\[
\frac{1}{\sqrt{f}} = 4.0 \left( \frac{1}{n} \right)^0.75 \log_10\left[ 10^{(3.5n+1)(1/2)} - 0.40 \right].
\]

These authors and others have demonstrated that these results can represent liquids with a variety of behavior over limited ranges by

**Turbulent Flow.** Correlations have been achieved for all four models, Eqs. (6.45)–(6.48). For power-law flow the correlation of Dodge and Metzner (1959) is shown in Figure 6.5(a) and is represented by the equation

\[
\frac{1}{\sqrt{f}} = 4.0 \left( \frac{1}{n} \right)^0.75 \log_10\left[ 10^{(3.5n+1)(1/2)} - 0.40 \right].
\]

These authors and others have demonstrated that these results can represent liquids with a variety of behavior over limited ranges by

**Table 6.7. Laminar Flow: Volumetric Flow Rate, Friction Factor, Reynolds Number, and Hedstrom Number**

<table>
<thead>
<tr>
<th>( D ) (cm)</th>
<th>( 10^{-4} ) He</th>
<th>( x_e )</th>
<th>( Re_{m5} )</th>
<th>( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.06</td>
<td>0.479</td>
<td>2.06</td>
<td>1635</td>
<td>114(120)</td>
</tr>
<tr>
<td>4.04</td>
<td>0.635</td>
<td>4.04</td>
<td>8946</td>
<td>93(109)</td>
</tr>
<tr>
<td>7.75</td>
<td>0.750</td>
<td>7.75</td>
<td>14,272</td>
<td>77</td>
</tr>
</tbody>
</table>

The Bingham data of Figure 6.6 are represented by the equations of Hanks [AIChE J. 9, 306 (1963)],

\[
(Re_p)_{m} = \frac{He}{8x_e} \left[ \left( 1 - \frac{4}{3} x_e + \frac{1}{3} x_e^2 \right) \right].
\]

(6.56)

\[
\frac{x_e}{He} = \frac{1}{16,800}.(6.57)
\]

They are employed in Example 6.10.

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**Examples 6.10**—(continued)

<table>
<thead>
<tr>
<th>( D ) (cm)</th>
<th>( 10^{-4} ) He</th>
<th>( x_e )</th>
<th>( Re_{m5} )</th>
<th>( V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.75</td>
<td>0.750</td>
<td>7.75</td>
<td>14,272</td>
<td>77</td>
</tr>
</tbody>
</table>

The numbers in parentheses correspond to the break points on the figure and agree roughly with the calculated values.

The solution of this problem is based on that of Wasp et al. (1977).
6.7. GASES

The differential energy balances of Eqs. (6.10) and (6.15) with the friction term of Eq. (6.18) can be integrated for compressible fluid flow under certain restrictions. Three cases of particular importance are of isentropic or isothermal or adiabatic flows. Equations will be developed for them for ideal gases, and the procedure for nonideal gases also will be indicated.

ISENTRROPIC FLOW

In short lines, nozzles, and orifices, friction and heat transfer may be neglected, which makes the flow essentially isentropic. Work transfer also is negligible in such equipment. The resulting theory is a basis of design of nozzles that will generate high velocity gases for power production with turbines. With the assumptions indicated, Eq. (6.10) becomes simply

\[ dH + (1/g_c)\mu du = 0, \]

which integrates into

\[ H_2 - H_1 + \frac{1}{2g_c}(u_2^2 - u_1^2) = 0. \]

One of these velocities may be eliminated with the mass balance,

\[ \dot{m} = u_2A_2/V_2 = u_1A_1/V_1 \]

so that

\[ u_2^2 - u_1^2 = (\dot{m}V_2/A_2)[1 - (A_1/V_1)^2]. \]

For ideal gases substitutions may be made from

\[ H_2 - H_1 = C_p(T_2 - T_1) \]